

Chiral loop corrections to weak decays of B mesons to positive and negative parity charmed mesons

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ABSTRACT: We determine chiral loop corrections to the B meson decay amplitudes to positive and negative parity charmed mesons within a framework which combines heavy quark and chiral symmetries. Then we investigate the impact of the lowest-lying positive parity heavy mesons on the determination of the Isgur-Wise functions. The corrections due to these states are competitive with the contributions arising from K and η meson loops. Since lattice studies rely on the chiral behavior of the amplitudes we discuss the chiral limit of our results. We find that the determination of the slope at zero recoil of the Isgur-Wise function ξ for the B transition to negative parity charm mesons is moderately affected by the inclusion of new states, while the slope of $\tau_{1/2}$ is affected significantly more.

KEYWORDS: Weak Decays, B-Physics, Chiral Lagrangians, Heavy Quark Physics.

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1. Introduction

The present experimental knowledge on the values of the CKM parameters is firmly established. The goal is to achieve highest possible accuracy in their experimental extraction. In order to reach it in exclusive decay modes one has to gain maximal control over the relevant form factors. In the case of precise determination of the V_{cb} CKM matrix elements the studies of B meson decays into charm resonances have been playing a prominent role. In experiments aimed to determine V_{cb} , actually the product $|V_{cb}\mathcal{F}(1)|$ is extracted, where $\mathcal{F}(1)$ is the $B \rightarrow D$ or $B \rightarrow D^*$ hadronic form factor at zero recoil. A lack of precise information about the shapes of various form factors is thus still the main source of uncertainties. In theoretical studies, heavy quark symmetry has been particularly appealing due to the reduction of six form factors in the case of $B \rightarrow D(D^*)l\nu_l$ transitions to only one [1, 2]. In addition, at zero recoil, when the final state meson is at rest in the B rest frame, the normalization of the form factors is fixed by symmetry. However, the results obtained within heavy meson effective theories obtain important corrections coming from operators which are suppressed as $1/M_{B,D}$ [3] as well as of higher order in the chiral expansion [4–7]. The knowledge of both kinds of corrections has improved during the last few years. The $B \rightarrow D^*l\nu_l$ decay amplitude is corrected by $1/M_{B,D}$ only at the second order in this expansion making it more appropriate for the experimental studies [6, 8]. In addition to heavy meson effective theory, other approaches have been used in the study of the $B \rightarrow D(D^*)$ form factors, such as quark models [9] and QCD sum rules [10], while the most reliable results should be expected from lattice QCD [11]. In the treatment of hadronic properties using lattice QCD the main problems arise due to the small masses of the light quarks. Namely, lattice studies have to consider light quarks with larger masses and then extrapolate results to their physical values. In these studies the chiral behavior of the amplitudes is particularly important. Heavy meson chiral perturbation theory

(HM χ PT) is very useful in giving us some control over the uncertainties appearing when the chiral limit is approached [12, 13]. Most recently in ref. [14], the authors have discussed $B \rightarrow D l \nu_l$ and $B \rightarrow D^* l \nu_l$ form factors in staggered chiral perturbation theory by including next-to-leading order corrections in staggered chiral perturbation theory.

The practitioners of the heavy meson effective theories faced new tasks when the charm mesons of the positive parity were discovered. In addition to understanding their structure, mass differences and decay properties, the fact that observed resonances lie only about 350 MeV above negative parity states stimulated many studies [15–17]. The inclusion of these states into the heavy meson effective theory was done fifteen years ago by including a number of unknown parameters [4, 5, 7] into the HM χ PT Lagrangian. Recently, in [12, 13] the role of positive parity states in the $B_{d,s}^0 - \bar{B}_{d,s}^0$ transitions and strong decays was investigated and a few important statements were reached: the contributions coming from positive parity states are competitive in size with the kaon and η meson loop corrections. However, they do not alter the pion chiral logarithms and consequently they provide a guideline for the lattice extrapolation of these results. A similar conclusion was already hinted on long time ago in the case of $B \rightarrow D^{(*)}$ form factors [5], although a complete analysis could not be performed at the time.

In this paper we reinvestigate chiral loop corrections within HM χ PT to the semileptonic transitions of B mesons into charm mesons of negative as well as positive parity. Specifically we study the effects of the small mass splitting between positive and negative parity heavy meson states on the leading non-analytic chiral behavior of the amplitudes. In section 1 we give the main details of our framework. Section 3 contains calculation of chiral loops while in section 4 we discuss chiral extrapolation. In section 5 we briefly summarize our results.

2. Framework

We use the formalism of heavy meson chiral Lagrangians [18, 19]. The octet of light pseudoscalar mesons can be encoded into $\Sigma = \xi^2 = \exp(2i\pi^i \lambda^i / f)$ where the $\pi^i \lambda^i$ matrix contains the pseudo-Goldstone fields

$$\pi^i \lambda^i = \begin{pmatrix} \frac{1}{\sqrt{6}}\eta + \frac{1}{\sqrt{2}}\pi^0 & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{6}}\eta - \frac{1}{\sqrt{2}}\pi^0 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \quad (2.1)$$

and $f \approx 120$ MeV at one loop [20]. The heavy-light mesons are customarily cataloged using the total angular momentum of the light degrees of freedom in the heavy meson j_ℓ^P which is a good quantum number in the heavy quark limit due to heavy quark spin symmetry. The negative $(j_\ell^P) = 1/2^-$ and positive $(j_\ell^P) = 1/2^+$ parity doublets can then be respectively represented by the fields $H(v) = 1/2(1 + \gamma \cdot v)[P_\mu^*(v)\gamma^\mu - P(v)\gamma_5]$, where $P_\mu^*(v)$ and $P(v)$ annihilate the vector and pseudoscalar mesons of velocity v , and $S(v) = 1/2(1 + \gamma \cdot v)[P_{1\mu}^*(v)\gamma^\mu\gamma_5 - P_0(v)]$ for the axial-vector ($P_{1\mu}^*(v)$) and scalar ($P_0(v)$) mesons.

The strong interactions Lagrangian relevant for our study of chiral corrections to processes among heavy mesons of velocity v is then at leading order in chiral and heavy quark expansion

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_\chi + \mathcal{L}_{\frac{1}{2}^-} + \mathcal{L}_{\frac{1}{2}^+} + \mathcal{L}_{\text{mix}}, \\
\mathcal{L}_\chi &= \frac{f^2}{8} \partial_\mu \Sigma_{ab} \partial^\mu \Sigma_{ba}^\dagger + \lambda_0 \left[(m_q)_{ab} \Sigma_{ba} + (m_q)_{ab} \Sigma_{ba}^\dagger \right], \\
\mathcal{L}_{\frac{1}{2}^-} &= -\text{Tr} \left[\overline{H}_a(v) (i v \cdot \mathcal{D}_{ab} - \delta_{ab} \Delta_H) H_b(v) \right] + g \text{Tr} \left[\overline{H}_b(v) H_a(v) \gamma \cdot \mathcal{A}_{ab} \gamma_5 \right], \\
\mathcal{L}_{\frac{1}{2}^+} &= \text{Tr} \left[\overline{S}_a(v) (i v \cdot \mathcal{D}_{ab} - \delta_{ab} \Delta_S) S_b(v) \right] + \tilde{g} \text{Tr} \left[\overline{S}_b(v) S_a(v) \gamma \cdot \mathcal{A}_{ab} \gamma_5 \right], \\
\mathcal{L}_{\text{mix}} &= h \text{Tr} \left[\overline{H}_b(v) S_a(v) \gamma \cdot \mathcal{A}_{ab} \gamma_5 \right] + \text{h.c.}
\end{aligned} \tag{2.2}$$

$\mathcal{D}_{ab}^\mu = \delta_{ab} \partial^\mu - \mathcal{V}_{ab}^\mu$ is the covariant heavy meson derivative. The light meson vector and axial currents are defined as $\mathcal{V}_\mu = 1/2(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)$ and $\mathcal{A}_\mu = i/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$ respectively. A trace is taken over spin matrices and the repeated light quark flavor indices. The λ_0 term induces masses of the pseudo-Goldstone mesons $m_{ab}^2 = 4\lambda_0(m_a + m_b)/f^2$. Accordingly \mathcal{L}_χ is of the order $\mathcal{O}(p^2)$ in the chiral power counting while the rest of this leading order Lagrangian is of the order $\mathcal{O}(p^1)$. Exceptions are the Δ_H and Δ_S residual masses of the H and S fields respectively. In a theory with only H fields, one is free to define the velocity v such that $\Delta_H = 0$ and all loop divergences are canceled by $\mathcal{O}(m_q)$ counterterms at zero order in $1/m_H$ expansion. However, once S fields are added to the theory, another dimensionful quantity $\Delta_{\text{SH}} = \Delta_S - \Delta_H$ enters loop calculations and does not vanish in the chiral and heavy quark limit [21]. We fix its value close to the phenomenological mass splitting between the even and odd parity heavy meson multiplets $\Delta_{\text{SH}} \approx 400$ MeV although smaller values have also been proposed when taking into account next to leading order terms in $1/m_H$ expansion [21]. For the couplings g , h and \tilde{g} we use the recently estimated values of [13] $g \simeq 0.6$, $h \simeq -0.5$ and $\tilde{g} \simeq -0.1$.

The weak part of the Lagrangian describing transitions among heavy quarks can be matched upon weak heavy quark currents in HQET [5, 19]

$$\begin{aligned}
\bar{c}_v \Gamma b_{v'} &\rightarrow C_{cb} \left\{ -\xi(w) \text{Tr} \left[\overline{H}_a(v) \Gamma H_a(v') \right] \right. \\
&\quad \left. -\tilde{\xi}(w) \text{Tr} \left[\overline{S}_a(v) \Gamma S_a(v') \right] - \tau_{1/2}(w) \text{Tr} \left[\overline{H}_a(v) \Gamma S_a(v') \right] + \text{h.c.} \right\}
\end{aligned} \tag{2.3}$$

at leading order in chiral and heavy quark expansion and where $\Gamma = \gamma_\mu(1 - \gamma_5)$ and $w = v \cdot v'$. Note that heavy quark symmetry dictates the values of $\xi(1) = \tilde{\xi}(1) = 1$, which should not receive any chiral corrections. On the other hand $\tau_{1/2}(w)$ is not constrained and we use the recently determined value of [22] $\tau_{1/2}(1) = 0.38$.

3. Calculation of chiral loop corrections

Here we present the most important details of our calculation of leading chiral loop corrections to the Isgur-Wise functions $\xi(w)$, $\tilde{\xi}(w)$ and $\tau_{1/2}(w)$. Following refs. [23, 24], we

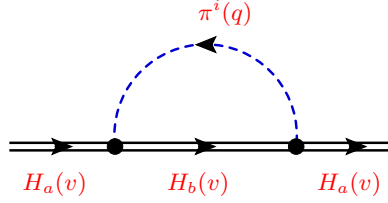


Figure 1: Self-energy ("sunrise" topology) diagram. Double lines represent heavy mesons, dashed lines represent pseudo-Goldstone bosons, while filled dots represent effective strong vertices.

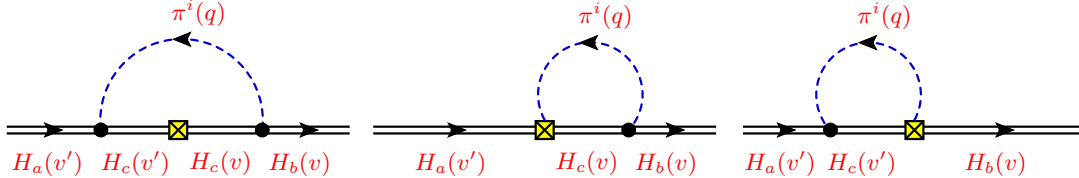


Figure 2: Weak vertex correction diagrams. Crossed boxes represent effective weak vertices. Only diagrams of the utmost left topology contribute to the amplitude at the leading chiral log order.

absorb the infinite and scale dependent pieces from one loop amplitudes into the appropriate counterterms at order $\mathcal{O}(m_q)$ (see e.g. [13]). We first calculate the wave function renormalization Z_{2H} of the heavy $H(v) = P(v)$, $P^*(v)$ and $P_0(v)$, $P_1^*(v)$ fields. This has been done e.g. in ref. [13] and we only quote the result at the $\mathcal{O}(p^2)$ power counting order (in appendix A). We get non-zero contributions to the heavy meson wavefunction renormalization from the self energy ("sunrise" topology) diagrams in figure 1 with leading order couplings in the loop.

In the case of the $P(v)$ mesons both vector $P^*(v)$ and scalar $P_0(v)$ mesons can contribute in the loop. The positive parity $P_0(v)$ and $P_1^*(v)$ similarly obtain wavefunction renormalization contributions from self energy diagrams (figure 1) with $P_1^*(v)$, $P(v)$ and $P_0(v)$, $P_1^*(v)$, $P^*(v)$ mesons in the loops respectively. Then we calculate loop corrections to the effective weak vertices. These come from the one loop diagram topologies shown in figure 2.

Namely, the initial and final heavy states may exchange a pseudo-Goldstone, while pairs of positive and negative parity heavy mesons may propagate in the loop (the left diagram in figure 2). Again not all heavy states contribute due to parity conservation in effective strong interaction vertices. Thus, when initial and final states are pseudoscalars we get contributions from pairs of $P^*(v')P^*(v)$, $P_0(v')P^*(v)$, $P^*(v')P_0(v)$ and $P_0(v')P_0(v)$ propagating in the loop, while for pseudoscalar initial and scalar final state we get contributions from pairs of $P^*(v')P(v)$, $P^*(v')P_1^*(v)$, $P_0(v')P(v)$ and $P_0(v')P_1^*(v)$ in the loop (due to heavy quark symmetry, the same results are obtained for (axial)vector external states, although different intermediate states contribute). On the other hand diagrams containing pseudo-Goldstone emission or absorption from the weak vertex (central and right diagram in figure 2) do not contribute at all at the leading order in the chiral power counting, since they may only appear via new weak operators containing derivatives or mass operators of

the pseudo-Goldstone fields. The complete expressions for the loop corrected $\xi(w)$, $\tilde{\xi}(w)$ and $\tau_{1/2}(w)$ we obtain are rather lengthy and can be found in appendix A.

4. Chiral extrapolation

We study the contributions of the additional resonances in the chiral loops to the chiral extrapolations employed by lattice QCD studies to run the light meson masses from the large values used in the simulations to the chiral limit [25, 26]. In order to tame the chiral behavior of the amplitudes containing the mass gap between the ground state and excited heavy meson states Δ_{SH} , we use the $1/\Delta_{\text{SH}}$ expansion of the chiral loop integrals [13]. Namely when Δ_{SH} is larger than the pion mass, we can expand the loop integrals and obtain a series in m_i/Δ_{SH} , where m_i are the pseudo-Goldstone masses. Since this presents an expansion around the decoupling limit — a theory with dynamical heavy meson states of only single parity — where the loops containing opposite parity states are replaced with effective vertexes, the resulting chiral logarithmic corrections contain no absorptive parts originating from the parity splitting¹. It turns out that actually all the zeroth order contributions vanish in this expansion and the presence of the nearby opposite parity states does not affect the leading pionic logarithmic behavior of the physical quantities. It does however affect the kaon and η -meson loops because those states are heavier than Δ_{SH} .

As argued in ref. [12] the $1/\Delta_{\text{SH}}$ expansion works well in an SU(2) theory where kaons and etas, whose masses would compete with the Δ_{SH} splitting, do not propagate in the loops. Therefore we write down explicit expressions for the chiral loop corrected Isgur-Wise functions specifically for the strangeless states ($a = u, d$) in the SU(2) theory:

$$\xi_{aa}(w) = \xi(w) \left\{ 1 + \frac{3}{32\pi^2 f^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} \left[g^2 2(r(w) - 1) - h^2 \frac{m_\pi^2}{4\Delta_{\text{SH}}^2} \left(1 - w \frac{\tilde{\xi}(w)}{\xi(w)} \right) - hg \frac{m_\pi^2}{\Delta_{\text{SH}}^2} w(w-1) \frac{\tau_{1/2}(w)}{\xi(w)} \right] \right\}, \quad (4.1)$$

and

$$\tau_{1/2aa}(w) = \tau_{1/2}(w) \left\{ 1 + \frac{3}{32\pi^2 f^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} \left[-g\tilde{g}(2r(w) - 1) - \frac{3}{2}(g^2 + \tilde{g}^2) + h^2 \frac{m_\pi^2}{4\Delta_{\text{SH}}^2} (w-1) - hg \frac{m_\pi^2}{2\Delta_{\text{SH}}^2} \frac{\xi(w)}{\tau_{1/2}(w)} w(1+w) + h\tilde{g} \frac{m_\pi^2}{2\Delta_{\text{SH}}^2} \frac{\tilde{\xi}(w)}{\tau_{1/2}(w)} w(1+w) \right] \right\}, \quad (4.2)$$

where

$$r(x) = \frac{\log(x + \sqrt{x^2 - 1})}{\sqrt{x^2 - 1}}, \quad (4.3)$$

¹Note that chiral corrections calculated in the full theory do contain absorptive parts as given in appendix A, when functions C_i are specified.

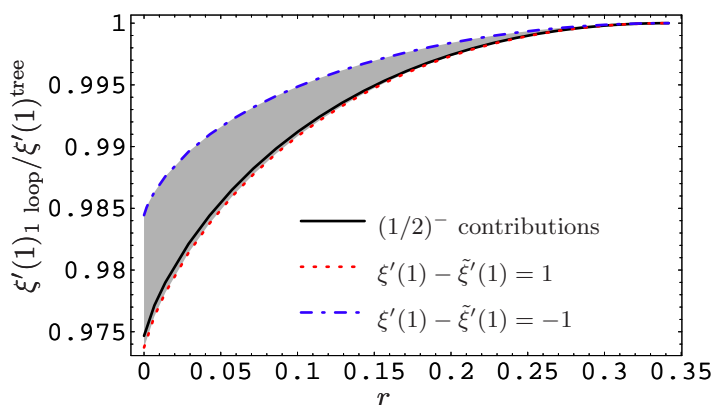


Figure 3: Chiral extrapolation of the slope of the IW function at $w = 1$ ($\xi'(1)$). Negative parity heavy states' contributions (black line) and a range of possible positive parity heavy states' contribution effects when the difference of slopes of $\xi(1)$ and $\tilde{\xi}(1)$ is varied between 1 (red dashed line) and -1 (blue dash-dotted line).

so that $r(1) = 1$ and $r'(1) = -1/3$. The first lines of eqs. (4.1) and (4.2) contain the leading contributions while the calculated $1/\Delta_{\text{SH}}$ corrections are contained in the second lines. Note that the positive parity heavy mesons contribute only at the $1/\Delta_{\text{SH}}^2$ order in this expansion since all the possible $1/\Delta_{\text{SH}}$ contributions vanish in dimensional regularization and the affected loop integral expressions have to be expanded up to the second order in $1/\Delta_{\text{SH}}$.

We then plot the chiral behavior of the Isgur-Wise function renormalization in the chiral limit below the Δ_{SH} scale in figures 3 and 4.

We have normalized the values of the extrapolated quantities at $m_\pi \sim \Delta_{\text{SH}}$ to 1 and perform the chiral extrapolation using the Gell-Mann - Okubo formulae as in [16] $m_\pi^2 = 8\lambda_0 m_s r / f^2$ where $r = m_{u,d} / m_s$ and $8\lambda_0 m_s / f^2 = 2m_K^2 - m_\pi^2 = 0.468 \text{ GeV}^2$. We limit our extrapolation to the region $m_\pi < \Delta_{\text{SH}}$ where the $1/\Delta_{\text{SH}}$ expansion is valid in a SU(2) theory. Presently no reliable estimates exist for the values of $\tilde{\xi}'(1)$ and $\tau'_{1/2}(1)$, which feature in chiral extrapolation involving opposite parity heavy states. Therefore we estimate their possible effects by varying their relative values in respect to $\xi'(1)$ between 1 and -1 in our extrapolations. We see that the effects of positive parity states' in the chiral loops on the chiral extrapolation of $\xi'(1)$ appear to be mild (around one percent in our estimate) below the Δ_{SH} scale (the gray shaded region around the leading order result in black solid line). Actually if $\xi'(1) - \tilde{\xi}'(1)$ is positive as reasoned in [5] and around 1, these leading $1/\Delta_{\text{SH}}$ corrections almost vanish. The same general chiral behavior can be attributed to $\tilde{\xi}'(1)$ with the substitutions $g \leftrightarrow \tilde{g}$, $\Delta_{\text{SH}} \leftrightarrow -\Delta_{\text{SH}}$ and $\xi'(1) \leftrightarrow \tilde{\xi}'(1)$. Also, the chiral extrapolation (including small leading $1/\Delta_{\text{SH}}$ contributions) of the $\tau_{1/2}(1)$ normalization appears fairly flat, indicating a linear extrapolation as a good approximation, whereas the effects of chiral loops on the extrapolation of its slope $\tau'_{1/2}(1)$ appear to be sizable, up to 30% in our crude estimate.

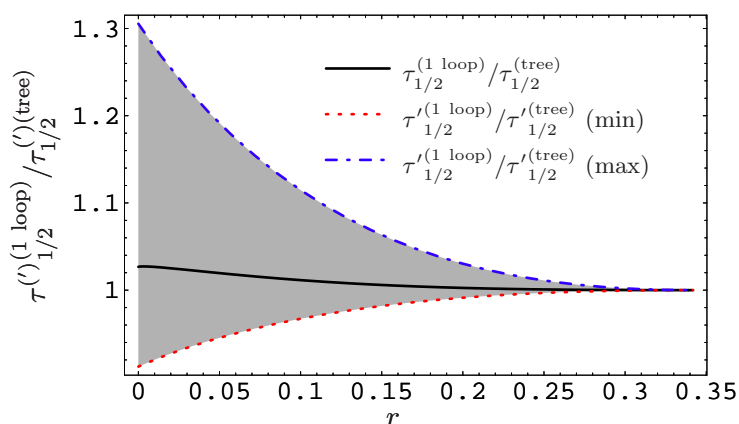


Figure 4: Chiral extrapolation of the $\tau_{1/2}$ function and its slope at $w = 1$. $\tau_{1/2}(1)$ extrapolation including $1/\Delta_{\text{SH}}$ contributions (black solid line), and a range of possible extrapolation effects of its slope — $\tau'_{1/2}(1)$ — (gray shaded region) when the difference of slopes $\xi'(1)$, $\tilde{\xi}'(1)$ and $\tau'_{1/2}(1)$ is varied between 1 (red dashed line) and -1 (blue dash-dotted line).

5. Discussion and conclusion

Within a $\text{HM}\chi\text{PT}$ framework, which includes even and odd parity heavy meson interactions with light pseudoscalars as pseudo-Goldstone bosons, we have calculated chiral loop corrections to the functions ξ and $\tau_{1/2}$. Motivated by the results of refs. [12, 13] where it was shown that the leading pionic chiral logarithms are not changed by the inclusion of even parity heavy meson states we consider chiral extrapolation of Isgur-Wise functions. Our analysis confirms that the form of the leading pionic logarithmic corrections to the Isgur-Wise functions is not changed by the inclusion opposite parity heavy mesons; they only contribute at the $m^4 \log m^2$ order as can be inferred by comparing eq. (4.1) with eq. (8) of ref. [7]. Our results are particularly important for the lattice QCD extraction of the form factors. The present errors on the V_{cb} parameter in the exclusive channels are of the order few percent. This calls for careful control over theoretical uncertainties in its extraction [27]. Our results on the chiral corrections are crucial in assuring validity of the form factor extraction and error estimation coming from the lattice studies. We find it especially important to stress that reliable chiral extrapolations can only be made in a $\text{SU}(2)$ limit with pion masses below the scale of Δ_{SH} . Our estimates for the leading $1/\Delta_{\text{SH}}$ corrections also constrain the accuracy of such extrapolations. On the other hand any ambiguities due to m_π dependence of the Δ_{SH} itself only become relevant at the next order in the chiral expansion. Note also that from our results given in appendix one can deduce the chiral corrections in the $B_s \rightarrow D_s$ decays which are not approached by experiment. Due to the strange quark flavor of final and initial heavy meson states, there is no leading pion logarithmic corrections making the lattice extraction below the heavy meson parity splitting gap Δ_{SH} much simpler.

In the $1/\Delta_{\text{SH}}$ expansion the opposite parity contributions yield formally next-to-leading chiral log order corrections in a theory with dynamical heavy meson fields of only single parity. Therefore they compete with $1/\Lambda_\chi$ corrections due to operators of higher chiral powers within chiral loops (yielding contributions such as those of the central and right diagrams in figure 2), where Λ_χ is the chiral symmetry breaking cut-off scale of the effective theory. In a theory containing propagating heavy meson states of both parities, the inclusion of such terms would in addition also yield $1/(\Lambda_\chi \Delta_{\text{SH}})$ terms. Our present approach to the estimation of the positive parity effects on the chiral extrapolation is therefore valid with the assumption $\Delta_{\text{SH}} < \Lambda_\chi$ where these additional contributions are further suppressed.

Acknowledgments

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A. Complete expressions of 1-loop corrected Isgur-Wise functions

Below are the complete expressions of the chiral 1-loop corrected Isgur-Wise functions, calculated as explained in the text. For the $\xi(w)$ we get

$$\begin{aligned} \xi_{ab}(w) = \xi(w) & \left\{ \delta_{ab} + \frac{1}{2} \delta Z_{2P_a(v')} + \frac{1}{2} \delta Z_{2P_b(v)} + \frac{\lambda_{ac}^i \lambda_{cb}^i}{16\pi^2 f^2} \right. & \text{(A.1)} \\ & \times \left[g^2 ((w+2)C_1(w, m, 0, 0) + (w^2 - 1)C_2(w, m, 0, 0)) \right. \\ & \quad \left. - h^2 \frac{\tilde{\xi}(w)}{\xi(w)} \left(\sum_{i=1}^4 C_i(w, m, \Delta_{\text{SH}}, \Delta_{\text{SH}}) \right. \right. \\ & \quad \quad \left. \left. + (w^2 - w + 1)C_2(w, m, \Delta_{\text{SH}}, \Delta_{\text{SH}}) \right) \right. \\ & \quad \left. - 2hg \frac{\tau_{1/2}(w)}{\xi(w)} (w-1) (C_1(w, m, \Delta_{\text{SH}}, 0) + wC_2(w, m, \Delta_{\text{SH}}, 0) \right. \\ & \quad \quad \left. \left. + C_4(w, m, \Delta_{\text{SH}}, 0) \right) \right] \Bigg\}, \end{aligned}$$

where the same formulae can be applied to $\tilde{\xi}(w)$ with the substitution $g \leftrightarrow \tilde{g}$ and $\Delta_{\text{SH}} \leftrightarrow$

$-\Delta_{\text{SH}}$. For the $\tau_{1/2}(w)$ on the other hand we obtain

$$\begin{aligned} \tau_{1/2ab}(w) = \tau_{1/2}(w) & \left\{ \delta_{ab} + \frac{1}{2}\delta Z_{2P_a(v')} + \frac{1}{2}\delta Z_{2P_{0b}(v)} + \frac{\lambda_{ac}^i \lambda_{cb}^i}{16\pi^2 f^2} \right. \\ & \times \left[g\tilde{g} \left((w-2)C_1(w, m, 0, 0) + (w^2-1)C_2(w, m, 0, 0) \right) \right. \\ & - h^2 \left(w \sum_{i=1}^4 C_i(w, m, \Delta_{\text{SH}}, -\Delta_{\text{SH}}) \right. \\ & \quad \left. \left. + (w^2 - w + 1)C_2(w, m, \Delta_{\text{SH}}, -\Delta_{\text{SH}}) \right) \right. \\ & + hg(w+1) \frac{\xi(w)}{\tau_{1/2}(w)} \left(C_1(w, m, 0, -\Delta_{\text{SH}}) \right. \\ & \quad \left. + wC_2(w, m, 0, -\Delta_{\text{SH}}) + C_3(w, m, 0, -\Delta_{\text{SH}}) \right) \\ & - h\tilde{g}(w+1) \frac{\tilde{\xi}(w)}{\tau_{1/2}(w)} \left(C_1(w, m, \Delta_{\text{SH}}, 0) \right. \\ & \quad \left. \left. + wC_2(w, m, \Delta_{\text{SH}}, 0) + C_4(w, m, \Delta_{\text{SH}}, 0) \right) \right] \left. \right\}. \end{aligned} \quad (\text{A.2})$$

In the above expressions $\delta Z_{2P} = (Z_{2P} - 1)$ are the chiral loop corrections to the heavy meson wavefunction renormalization:

$$Z_{2P_a(v)} = 1 - \frac{\lambda_{ab}^i \lambda_{ba}^i}{16\pi^2 f^2} \left[3g^2 C_1'(0, m_i) - h^2 C' \left(\frac{\Delta_{\text{SH}}}{m_i}, m_i \right) \right], \quad (\text{A.3})$$

for the negative parity doublet and

$$Z_{2P_{0a}(v)} = 1 - \frac{\lambda_{ab}^i \lambda_{ba}^i}{16\pi^2 f^2} \left[3\tilde{g}^2 C_1'(0, m_i) - h^2 C' \left(-\frac{\Delta_{\text{SH}}}{m_i}, m_i \right) \right], \quad (\text{A.4})$$

for the positive parity states. As in ref. [24], a trace is assumed over the inner repeated index(es) (here b).

We make use of the C_i loop integral functions, of which $C_i(x, m)$ have been defined in ref. [13], while $C_i(w, m, \Delta_1, \Delta_2)$ have been defined in [3]. The $1/\Delta$ expansion of $C_i(x, m)$ has been demonstrated in ref. [13] while for $C_i(w, m, \Delta_1, \Delta_2)$ it follows as

$$\begin{aligned} C_1(w, m, \Delta, 0) &= C_1(w, m, 0, \Delta) \rightarrow -(1/\Delta)C_1(m, 0) - (1/\Delta^2)C_0(m)w + \mathcal{O}(1/\Delta^3), \\ C_2(w, m, \Delta, 0) &= C_2(w, m, 0, \Delta) \rightarrow -(1/\Delta^2)C_0(m) + \mathcal{O}(1/\Delta^3), \\ C_3(w, m, \Delta, 0) &= C_4(w, m, 0, \Delta) \rightarrow -(1/\Delta)C_1(m, 0) + (2/\Delta^2)C_0(m)w + \mathcal{O}(1/\Delta^3), \\ C_4(w, m, \Delta, 0), C_3(w, m, 0, \Delta) &\rightarrow \mathcal{O}(1/\Delta^3), \\ C_1(w, m, \Delta, \Delta) &= -C_1(w, m, \Delta, -\Delta) \rightarrow (1/\Delta^2)C_0(m) + \mathcal{O}(1/\Delta^3), \\ C_2(w, m, \Delta, \Delta), C_2(w, m, \Delta, -\Delta) &\rightarrow \mathcal{O}(1/\Delta^3), \\ C_3(w, m, \Delta, \Delta), C_3(w, m, \Delta, -\Delta) &\rightarrow \mathcal{O}(1/\Delta^3), \\ C_4(w, m, \Delta, \Delta), C_4(w, m, \Delta, -\Delta) &\rightarrow \mathcal{O}(1/\Delta^3), \end{aligned} \quad (\text{A.5})$$

where

$$C_0(m) = -\frac{1}{4}m^4 \log\left(\frac{m^2}{\mu^2}\right). \quad (\text{A.6})$$

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